

Performance Assessment Task
The Baker Grade 4
The task challenges a student to demonstrate understanding of the concepts involved in multiplication and division. A student must make sense of the relationship between multiplication and division. A student must understand division as the inverse of multiplication, the operation of sharing (partitive), partitioning (measurement), and repeated subtraction. A student must be able to recognize and use the factors of a given quantity. A student must determine a product and a quotient. A student must make sense of a division contextual problem with a remainder and without a remainder.
Common Core State Standards Math - Content Standards
<p><u>Numbers and Operations in Base Ten</u></p> <p>Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p><u>Operations and Algebraic Thinking</u></p> <p>Use the four operations with whole numbers to solve problems.</p> <p>4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p> <p>MP.6 Attend to precision.</p> <p>Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of</p>

precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
4	2007	10	5	77 %

The Baker

This problem gives you the chance to:

- choose and perform number operations in a practical context
-

The baker uses boxes of different sizes to carry her goods.



Cookie boxes hold 12 cookies.

Donut boxes hold 4 donuts.

Muffin boxes hold 2 muffins.

Bagel boxes hold 6 bagels.

Bagel boxes hold 4

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes _____

donut boxes _____

muffin boxes _____

bagel boxes _____

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make? _____

Show your calculations.

3. On Wednesday she baked 42 cookies.

How many boxes did she fill? _____

How many cookies were left over? _____

Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? _____

Show how you figured this out.

The Baker

Work the task. Look at the rubric. How does this task assess the big ideas about understanding the operation of multiplication and division?

Look at student work on part 1.

- How many of your students understood that this was a division situation?
- How many of your students just repeated the information in table (12,4,2,6)?
- How many multiplied by 24? (288, 48, 96,144)?
- Used addition? (36, 28, 26, 30)?
- Reversed answers for muffins and donuts (working down instead of across)?

What other types of errors did you notice?

How often do students get opportunities to work with division in context? How does context help them develop an understanding of the operation of division and its purpose?

What models do you use with students to help them understand the operation of multiplication and division?

Now look at work for part 3.

How many of your students could solve the problem correctly? What strategies did they use and what explanations did you find of value? How could you use some of these explanations as models with the class?

How many of your students thought 4 boxes would be filled with 6 remaining?

Think about the action of the story – 42 cookies being distributed into boxes. Why does “48 – 6” not make sense? The answer is 42, so where is the error? What is the important mathematical idea that students are missing? What kind of activity could you design or discussion question could you pose to help them think more deeply about the situation and examine the error in their computations?

What were some of the other answers that did not fit the action of the story? Why do you think students did not recognize this as a division situation? How do students learn the meaning of operations so that they can apply them appropriately rather than just randomly guessing an operation? What kinds of discussions help develop the desired sense-making of operations?

Looking at Student Work on The Baker

Student A is comfortable with the operation of division. The student shows work for all the parts of the task. The explanation in part 3 just gives a set of procedures, but does not explain why it makes sense. *While this get full marks, what types of explanations would you like students to strive for?*

Student A

Bagel boxes hold 6 bagels.

1. On Monday she baked 24 of everything.
How many boxes did she need? Fill in the empty spaces.

$$\begin{array}{r} 2 \\ 12 \overline{) 24} \\ \underline{24} \\ 0 \end{array}$$
 cookie boxes 2 ✓

$$\begin{array}{r} 6 \\ 4 \overline{) 24} \\ \underline{24} \\ 0 \end{array}$$
 donut boxes 6 ✓

 4 4

$$\begin{array}{r} 12 \\ 2 \overline{) 24} \\ \underline{24} \\ 0 \end{array}$$
 muffin boxes 12 ✓

$$\begin{array}{r} 4 \\ 6 \overline{) 24} \\ \underline{24} \\ 0 \end{array}$$
 bagel boxes 4 ✓

2. On Tuesday she baked just bagels. She filled 7 boxes.
How many bagels did she make? 42 ✓
Show your calculations.

$$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$$

$$\begin{array}{r} 3R6 \\ 12 \overline{) 42} \\ \underline{36} \\ 6 \end{array}$$

$$\begin{array}{r} 12 \\ \times 4 \\ \hline 48 \end{array}$$

 1 1
1 1

3. On Wednesday she baked 42 cookies.
How many boxes did she fill? 3 ✓
How many cookies were left over? 6 ✓
Explain how you figured this out.
First I did $12 \overline{) 42}$ and got $3R6$.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.
What did she bake on Thursday? donuts ✓
Show how you figured this out.

$$\begin{array}{r} 8 \\ 4 \overline{) 32} \\ \underline{32} \\ 0 \end{array}$$

 10

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The Baker Test 4

Student B is starting to make sense of dimensional analysis and has clear labels for the work in part 2 and 4. While the student does computations that yield an answer of 42 in part 3, the strategy does not make sense in this context. *How could you help this student to understand her mistake? What question would you pose? How could you facilitate a class discussion around this solution to bring out the big mathematical ideas?*

Student B

1. On Monday she baked 24 of everything.
 How many boxes did she need? Fill in the empty spaces.

$$\begin{array}{r} 12 \\ +12 \\ \hline 24 \end{array}$$
 cookie boxes 2 ✓ donut boxes 6 ✓ $4 \times 6 = 24$ 4
 muffin boxes 12 ✓ $24 \div 2 = 12$ bagel boxes 4 boxes ✓ $6 \times 4 = 24$ 4

2. On Tuesday she baked just bagels. She filled 7 boxes.
 How many bagels did she make? 42 bagels ✓ 1
 Show your calculations.
~~4 boxes~~ \times ~~6 bagels~~ = ~~24~~ 7 boxes \times 6 bagels in each box = 42 bagels in all 1

$$\begin{array}{r} 4 \\ \times 6 \\ \hline 24 \end{array}$$

3. On Wednesday she baked 42 cookies.
 How many boxes did she fill? 4 boxes \times 12 12 0 0
 How many cookies were left over? 6 cookies ✓ $\times 2$ $\times 4$
 Explain how you figured this out.

$$\begin{array}{r} 12 \\ \times 2 \\ \hline 24 \\ - 48 \\ \hline 42 \end{array}$$

I figured this out by multiplying 4 and 12 10
then the answer was 48. I subtracted 48 with 1
his and I got 42. ✓

4. On Thursday she baked 32 of just one item and she filled 8 boxes.
 What did she bake on Thursday? muffins \times 0
 Show how you figured this out.
 $32 \div 4 = 8$ $\begin{array}{r} 4 \\ \times 8 \\ \hline 32 \end{array}$ 4 muffins 1
 $\times 8$ boxes 1
32 in total The Baker Test 4 10
 7
 \times
 0
 1
 1
 1

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Now look at the explanation for part 3 of Student C. Notice how the student clearly describes the purpose of the calculations and how they relate to the situation. Although the student uses multiplication to solve the problems, the student's grasp on the situation and underlying meaning of multiplication/division is evident.

Student C

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes 2 ✓ donut boxes 6 ✓
 muffin boxes 12 ✓ bagel boxes 4 ✓ 44

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?

Show your calculations.

42 Bagels ✓

$$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$$

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?

How many cookies were left over?

Explain how you figured this out.

$$\begin{array}{r} 3 \text{ boxes} \\ \times 12 \\ \hline 36 \end{array}$$

$$\begin{array}{r} 42 \\ - 36 \\ \hline 6 \end{array}$$

4 boxes would be over 42 cookies. 3 boxes would use 36 cookies then I subtract 36 from 42 to find how many cookies were left over.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday?

Show how you figured this out.

She made donuts ✓

$$\begin{array}{r} 4 \\ \times 8 \\ \hline 32 \end{array}$$

Student D does not understand division. The student uses multiplication in part 1. The student did a drawing and counting strategy for part 3. In part 4 the student has to use repeated addition to find the solution. *How does context help students to build an understanding of operation? How might labels help the student to make sense of the situation? What question might you pose?*

Student D

1. On Monday she baked 24 of everything.
 How many boxes did she need? Fill in the empty spaces.

$$\begin{array}{r} \times 12 \\ 24 \\ \hline 48 \\ + 240 \\ \hline 288 \end{array}$$
 = cookie boxes 288 \times

$$\begin{array}{r} 24 \\ \times 4 \\ \hline 96 \end{array}$$
 = donut boxes 96 \times

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \end{array}$$
 = muffin boxes 48 \times

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \end{array}$$
 = bagel boxes 144 \times

2. On Tuesday she baked just bagels. She filled 7 boxes.
 How many bagels did she make? 42 ✓
 Show your calculations.

$$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array}$$
 $6 \times 7 = 42$ ✓

3. On Wednesday she baked 42 cookies.
 How many boxes did she fill? 10 \times
 How many cookies were left over? 2 \times
 Explain how you figured this out.

I figured it out by making 12 circles and putting talies on each one and it feald to 6 boxes and left 2 cookie boxes half way.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.
 What did she bake on Thursday? donut ✓
 Show how you figured this out.

4, 8, 12, 16, 20, 24, 28, 32. I counted by 4 to find the answer of how much she baked on Thursday.

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Student E uses an alternate division algorithm in part 3 and 4, which would have yielded the correct solution except for a subtraction slip. *Can you understand this method? Could you use it to solve another problem? How does it compare to the standard algorithm?*

Student E

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes 2 boxes ✓

donut boxes 6 boxes

muffin boxes 12 boxes ✓

bagel boxes 4 boxes

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make? 42 bagels ✓

Show your calculations.

$$\begin{array}{r} \times 7 \\ 6 \\ \hline 42 \end{array}$$

3. On Wednesday she baked 42 cookies.

How many boxes did she fill? 3 boxes ✓

How many cookies were left over? 4 cookies ✗

Explain how you figured this out.

$$\begin{array}{r} 12 \overline{) 42} \\ \underline{24} \\ 18 \\ \underline{12} \\ 4 \end{array}$$

I divided the 42 cookies with 12 cookies because that fits on a box and my answer was 3 R. 4

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? Donuts ✓

Show how you figured this out.

$$\begin{array}{r} 8 \overline{) 32} \\ \underline{32} \\ 0 \end{array}$$

4 10

Students, who were not comfortable with division, had other strategies to make sense of the situation. See the work of Student F.

Student F

1. On Monday she baked 24 of everything.
 How many boxes did she need? Fill in the empty spaces.

cookie boxes 2 ✓ donut boxes 6 ✓
 muffin boxes 12 ✓ bagel boxes 4 ✓

2. On Tuesday she baked just bagels. She filled 7 boxes. ✓
 How many bagels did she make? 42
 Show your calculations.

$7 \times 6 = 42$

3. On Wednesday she baked 42 cookies.
 How many boxes did she fill? 3 ✓
 How many cookies were left over? 6 ✓

Explain how you figured this out.
 $42 - 36 = 6$

4. On Thursday she baked 32 of just one item and she filled 8 boxes. ✓
 What did she bake on Thursday? Donuts ✓
 Show how you figured this out.

$8 \times 4 = 32$

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Some students did not recognize filling boxes as a distributing or division action. Student G adds all the amounts together and finds that they total 24.

Student G

The baker uses boxes of different sizes to carry her goods.



Cookie boxes hold 12 cookies.
Donut boxes hold 4 donuts.
Muffin boxes hold 2 muffins.
Bagel boxes hold 6 bagels.

1. On Monday she baked 24 of everything.
 How many boxes did she need? Fill in the empty spaces.

cookie boxes 1 x donut boxes 1 x
 muffin boxes 1 x bagel boxes 1 x

$12 + 4 + 2 + 6 = 24$

Student H is still not comfortable with multiplication and division and uses repeated addition for everything. *How do we help students move to more efficient strategies?* Notice that in part 1 the student does not understand the operation needed and repeats the information in the table. In part 3 the student has a strategy that yields the correct solution, but the student doesn't recognize which part represents the remainder. *What might be good next steps for this student?*

Student H

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes 12^x cookies

donut boxes 4^x donuts

muffin boxes 2^x muffins

bagel boxes 6^x bagels

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make? 42^x bagels

Show your calculations.

$$6 + 6 + 6 + 6 + 6 + 6 + 6 =$$

3. On Wednesday she baked 42 cookies.

How many boxes did she fill? 3^x boxes

How many cookies were left over? 3^x cookies

Explain how you figured this out.

$$12 + 12 + 12 + 6 = 42 \text{ so } 42 \div 12 = 3$$

So 3 are left over. I multiplied to get the answer for how many boxes she filled?

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? Donuts

Show how you figured this out.

$$4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 32$$

Student I understands that the context is a division operation. It is unclear about whether the student can do the arithmetic of division. The technology of the calculator obscures some of the information of the task. The student doesn't understand that the 0.5 means half of the group of twelve or 6. The mathematical understanding necessary to use a calculator in this task might be more complicated than understanding the arithmetic.

Student I

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes 2

donut boxes 6

muffin boxes 12

bagel boxes 4

44

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?

42

Show your calculations.

boxes 11
 *6 bagels in 11
 each box
 42 bagels in all

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?

3

How many cookies were left over?

54

Explain how you figured this out.

On the calculator I did $42 \div 12 = 3.5$
 which is 3RS.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday?

Donuts

Show how you figured this out.

$\frac{32}{8} = 4$
 $\frac{32}{5.3333333}$
 10

While Student J received 6 points, there are some alarming ideas in the work. Notice that in part one, the student was unclear about operations and tried a variety of strategies to get the answer. In part two the student did a drawing and counting strategy. *Do you think this was an attempt to please the teacher (show the work) or do you think the student needed to think about what the answer would be?* The student doesn't recognize the operations in part 3. Fill means add and left means subtract. Notice that the answer in part 4 is the logic of elimination rather than computation.

Student J

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes ~~2~~ ✓ $\begin{array}{r} 24 \\ + 12 \\ \hline 36 \\ + 24 \\ \hline 60 \end{array}$ donut boxes ~~6~~ ✓ $\begin{array}{r} 24 \\ + 4 \\ \hline 28 \\ + 24 \\ \hline 52 \end{array}$

muffin boxes ~~20~~ ✓ $\begin{array}{r} 24 \\ + 12 \\ \hline 36 \\ + 24 \\ \hline 60 \end{array}$ bagel boxes ~~4~~ ✓ $\begin{array}{r} 24 \\ + 4 \\ \hline 28 \\ + 24 \\ \hline 52 \end{array}$

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make? 42 ✓

Show your calculations.

 $6 \times 7 = 42$ 2 ✓

3. On Wednesday she baked 42 cookies.

How many boxes did she fill? $\begin{array}{r} 42 \\ + 12 \\ \hline 54 \end{array}$ X

How many cookies were left over? $\begin{array}{r} 42 \\ - 12 \\ \hline 30 \end{array}$ X

Explain how you figured this out.

I added how many cookies she made and how much did she fill. In the other I subtracted cause it said how many were left.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? She baked Donuts ✓ 11

Show how you figured this out.

Cause it did not talk about Donuts. X 0 0

Student K also struggles with operations. The student doesn't understand division in part 2. *Why doesn't 28 make sense in this situation? Where do you think the 28 comes from?* In part 4 the student interprets fill as multiply and left as subtract. *What questions would you like to ask the student?*

Student K

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes 2 ✓

donut boxes 6 ✓

muffin boxes 12 ✓

bagel boxes 4 ✓

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?

28 $\overset{28}{\div} 24$ X

Show your calculations.

X 9
0

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?

504 X
462

How many cookies were left over?

Explain how you figured this out.

+242 4504
x12 SAT 462 X

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday?

muffins X

Show how you figured this out.

6
0
4

Now look at the work of Student L. Notice that in part one the student multiplies. In part 2 the student attempts a drawing strategy, but doesn't even make the amount the same in each box. Understanding "equal-size groups" is fundamental to all work in multiplication, division, and proportional reasoning. In part 4 the student multiplies and then subtracts. Where does the "-201" come from? This student is obviously missing some very important background bits in working with numbers, although the student can do multiplication and seems to know multiplication facts (the carry the one indicates the student wasn't using a calculator). *How do we help students understand the meaning of operations? How is this different from learning facts and algorithms?*

Student L

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

cookie boxes 96 X

donut boxes 96 X

muffin boxes 48 X

bagel boxes 144 X

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make? 86 X

Show your calculations.



3. On Wednesday she baked 42 cookies.

How many boxes did she fill? 12 X

How many cookies were left over? 1 X

Explain how you figured this out.

because if she had 6 cookies it would hold
another six cookies it will hold one box. X

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? muffins X

Show how you figured this out.

$$\begin{array}{r}
 32 \\
 \times 3 \\
 \hline
 256 \\
 -201 \\
 \hline
 + 55 = 08
 \end{array}$$

Student Task	Choose and perform number operations in a practical context. Use multiplicative thinking and grouping to make sense of a context.
Core Idea 2 Number Operations	<p>Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.</p> <ul style="list-style-type: none"> • Understand division as the inverse operation of multiplication, the operation of sharing (partitive), partitioning (measurement), repeated subtraction, and an operation to determine rates. • Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems. • Develop fluency in multiplying whole numbers.

Based on teacher observation, this is what fourth graders know and are able to do

- Use division to find the number of boxes for 24 of something
- Multiply to find the number of bagels needed to fill 7 boxes

Areas of difficulty for fourth graders:

- Interpreting division with a remainder
- Understanding that while $(4 \times 12) - 6 = (3 \times 12) + 6$, the meaning in context is not the same
- Choosing appropriate operations

Strategies used by successful students:

- Using more than one division algorithm
- Using the relationship between multiplication and division to figure out number of groups
- Using repeated addition for multiplication and division
- Using labels to make sense of calculations

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 4

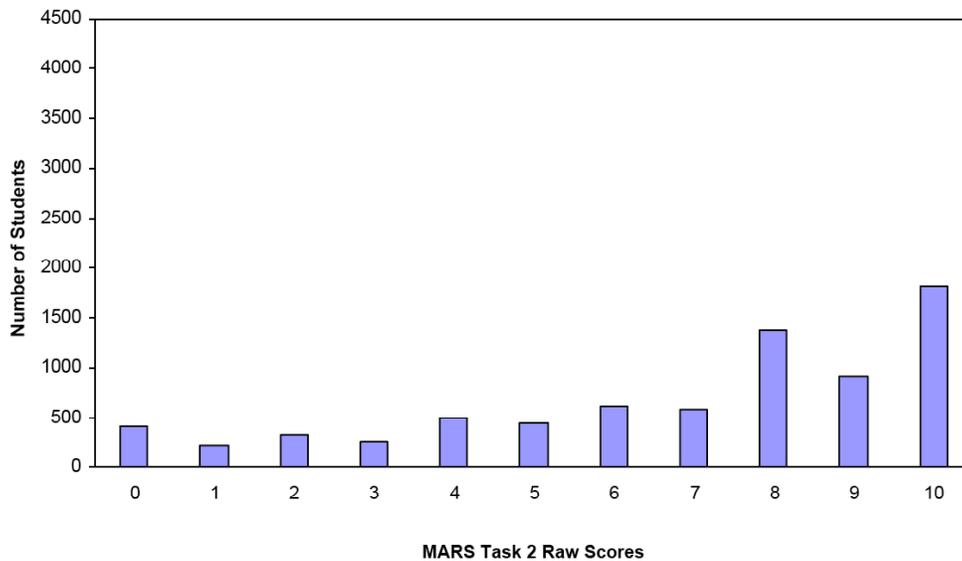
Task 2 - The Baker

Mean: 6.83 StdDev: 3.04

Table 21: Frequency Distribution of MARS Test Task 2, Grade 4

Task 2 Scores	Student Count	% at or below	% at or above
0	413	5.5%	100.0%
1	226	8.6%	94.5%
2	330	13.0%	91.4%
3	252	16.4%	87.0%
4	496	23.0%	83.6%
5	440	28.9%	77.0%
6	610	37.1%	71.1%
7	583	45.0%	62.9%
8	1373	63.4%	55.0%
9	914	75.7%	36.6%
10	1814	100.0%	24.3%

Figure 30: Bar Graph of MARS Test Task 2 Raw Scores, Grade 4



*The maximum score available on this task is 10 points.
The minimum score for a level 3 response, meeting standards, is 5 points.*

Most students, 90%, could use multiplication to find the number of bagels needed to fill 7 boxes. Many students, 84%, could find the number of bagels to fill 7 boxes, and could use division to find 32 of what items would fill 8 boxes. 77% could find out how many boxes of cookies, donuts, and muffins 24 would fill. More than half the students could solve all parts of the task except part 3, involving division with a remainder. 24% could meet all the demands of the task. Almost 6% scored no points on the task. All the students in the sample with this score attempted the task.

The Baker

Points	Understandings	Misunderstandings
0	All the students in the sample attempted the task.	Students had difficulty choosing operations. Many used subtraction or addition instead of multiplication and division.
1	Students could solve part 4, finding that it would take 32 donuts to fill 8 boxes.	10.5% of the students chose donuts, but showed no work. 4% thought the answer was muffins.
2	Students could find the number of bagels needed to fill 7 boxes and show their work.	4% thought there would be 14. 2% thought 13 were needed. 7% had answers higher than 50.
4	Students could solve part 2 and 4 of the task and show their work.	Students had trouble interpreting part 1. 10.5% just repeated the given information (12 cookies). 9% multiplied (4×24) instead of dividing ($24/4$). 4% added ($24 + 12$).
5	Students with this score could usually do 3 of the items in part 1 and find the answer with no work in part 4.	
6		Most students with this score missed all of part 1, dividing 24 to find the number of boxes.
8		Most students with this score missed all of part 3, interpreting division with a remainder. 14% of the students thought that 42 cookies could fill 4 boxes with 6 cookies left over ($4 \times 12 - 6$).
10	Students could recognize multiplication and division contexts and solve problems, including interpreting remainders in division. Students with this score seemed to understand the relationship between operations and could use multiplication, division, or repeated addition to get their answers.	

Implications for Instruction

Students need to do work with division in context to help them understand the meaning of division, making and counting equal size groups. In this task, many students could write number sentences that could yield a correct solution, $(4 \times 12) - 6 = 42$, but which did not fit the action of the story or yield the needed information of 3 groups of 12 with 6 remaining. Context provides for the backdrop for struggling with sense-making: What do I know? What do I need to find out? In this situation, how do I interpret the remainder? These are important ideas around division that are not focused on when just practicing procedures.

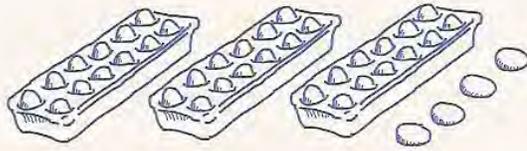
Models are useful tools to help students with understanding the meaning of operation. Division has two distinct meanings: a partitive model – sharing things or meting things out into equal groups and a measurement model – measuring how many equal size groups can be measured or made from a given quantity. The bar models are helpful tools to make students visualize the division situations.

Interpretation	Interpretive question	Diagram
Partitive division:	20 is 4 groups of what unit?	
Measurement division:	20 is how many groups of 4?	

Can you think of a word problem that would go with each of the models above? The division in this task is making sense of measurement division: how many groups or boxes of a certain item. Students are generally most familiar with partitive division. I have so many candies to share equally with my friends. Students need to be comfortable with a variety of types of division problems.

Below are some models for interpreting remainders:

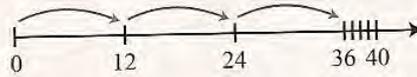
40 eggs is how many dozen?



3 groups with 4 left over.

$$40 \div 12 = 3 \text{ R } 4$$

How many feet in 40 inches?



Three feet with 4 inches left.

$$40 \div 12 = 3 \text{ R } 4$$

Ideas for Action Research – Exploring Models with Students

Think about problems that use different types of division. For example, consider these two problems:

5 cases of peaches weigh 450 lb. How much does each case weigh?

Jason's mom made 210 cupcakes for the school bake sale. She puts them into boxes of 10 each. How many boxes of cupcakes were there?

What would the bar models for each of these look like? How are the models different?

Introduce your students to the bar models and see what students think of the models. Do they find them interesting? How quickly do they make sense of models? What evidence do you see of changes in student thinking after being introduced to the bar models?

The bar models help students in a variety of ways for learning multiplication and division; and, they set the stage for thinking about proportional reasoning and understanding variables at later grade levels. For more examples of bar models, see Singapore math books 3A, 4A, and 5A (available from www.Singaporemath.com).